

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 23	
(a) $P(X > 70) = \frac{1}{2}$	MA-S3 Random Variables MA12-8, MA12-9 Band 4 • Gives the correct solution 1
(b) $P(49 \leq X \leq 77) = \int_{49}^{77} \frac{1}{\sqrt{2\pi(49)}} e^{-\frac{(x-70)^2}{2(49)}} dx$ $= \frac{1}{\sqrt{98\pi}} \int_{49}^{77} e^{-\frac{(x-70)^2}{98}} dx$ $\approx \frac{1}{\sqrt{98\pi}} \frac{77-49}{2 \times 4} \left[e^{-\frac{(49-70)^2}{98}} + 2 \left(e^{-\frac{(56-70)^2}{98}} \right. \right.$ $\left. \left. + e^{-\frac{(63-70)^2}{98}} + e^{-\frac{(70-70)^2}{98}} \right) + e^{-\frac{(77-70)^2}{98}} \right]$ $= \frac{1}{2\sqrt{2}\pi} \left[e^{-\frac{9}{2}} + 2 \left(e^{-2} + e^{-\frac{1}{2}} + 1 \right) + e^{-\frac{1}{2}} \right]$ $\approx 0.8181 \quad (\text{to four decimal places})$ <p>The probability required is the probability that a mark lies within three standard deviations to the left of the mean, and one standard deviations to the right of the mean. From the empirical rule, we know that approximately $\frac{99.7\%}{2} + \frac{68\%}{2} = 83.85\%$ of all marks lie in this range. Our approximation reflects this well, noting that we have only used four sub-intervals and hence cannot expect an extremely precise answer.</p> <p><i>Note: This question is more easily handled by first standardising the required probability. That is, set $Z = \frac{X-70}{7}$ and observe that $Z \sim N(0, 1)$. Accept responses that correctly use this approach.</i></p>	MA-C4 Integral Calculus MA-S3 Random Variables MA12-7, 8, 9, 10 Bands 4-6 • Gives correct approximation. AND • Gives a valid interpretation 4 OR • Gives correct approximation. • Gives a valid interpretation 3 OR • Appropriately uses the trapezoidal rule. OR • States integral AND links to empirical rule 2 • States the required integral 1
(c) Y <i>Note: The local maximum of a normal probability density function occurs at its mean.</i>	MA-S3 Random Variables MA12-8 Band 3 • Gives the correct solution 1
(d) $z_{\text{English}} = \frac{76-57}{17}$ $= 1.11764$ $z_{\text{Mathematics}} = \frac{76-70}{7}$ $= 0.85714$ <p>Comparing these z-scores, the student performed better relative to their English cohort.</p>	MA-S3 Random Variables MA12-8, MA12-9 Band 3 • Gives the correct answer AND computes TWO z-scores 2 • Computes ONE z-scores 1