Question 10 (7 marks)

a. Attempt product rule differentiation on both components to find $\dot{\underset{\sim}{r}}(t)$. M1

Product rule on the $\underset{\sim}{i}$ component:

$$\frac{d}{dt}(e^t\cos(t)) = -e^t\sin(t) + e^t\cos(t)$$

Product rule on the $\underset{\sim}{j}$ component:

$$\frac{d}{dt}(e^t\sin(t)) = e^t\cos(t) + e^t\sin(t)$$

$$\dot{\underset{\sim}{r}}(t) = (e^t\cos(t) - e^t\sin(t))\underset{\sim}{i} + (e^t\cos(t) + e^t\sin(t))\underset{\sim}{j}$$ A1

$$\dot{\underset{\sim}{r}}(0) = \underset{\sim}{i} + \underset{\sim}{j}$$ A1

b. $|\underset{\sim}{r}(t)| = e^t\sqrt{\cos^2(t) + \sin^2(t)}$

$= e^t$ A1

$|\dot{\underset{\sim}{r}}(t)| = e^t\sqrt{(\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2}$

$= e^t\sqrt{2\cos^2(t) + 2\sin^2(t)}$

$= \sqrt{2}e^t$ A1

Attempt to find $\underset{\sim}{r}(t) \cdot \dot{\underset{\sim}{r}}(t)$. M1

$\underset{\sim}{r}(t) \cdot \dot{\underset{\sim}{r}}(t) = e^{2t}(\cos(t)(\cos(t) - \sin(t)) + \sin(t)(\sin(t) + \cos(t)))$

$= e^{2t}(\cos^2(t) + \sin^2(t))$

$= e^{2t}$

Use of $\cos(\theta) = \dfrac{\underset{\sim}{r}(t) \cdot \dot{\underset{\sim}{r}}(t)}{|\underset{\sim}{r}(t)||\dot{\underset{\sim}{r}}(t)|}$ gives $\cos(\theta) = \dfrac{e^{2t}}{e^t \times \sqrt{2}e^t}$.

So $\cos(\theta) = \dfrac{1}{\sqrt{2}}$. A1

Hence, $\theta = \dfrac{\pi}{4}$ and $\underset{\sim}{r}(t)$ always makes an angle of $\dfrac{\pi}{4}$ with $\dot{\underset{\sim}{r}}(t)$.

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