

**SECTION II****60 marks****Attempt Questions 11–14****Allow about 1 hour and 45 minutes for this section**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

**Question 11** (15 marks) Use a SEPARATE writing booklet.

- (a) The polynomial  $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$  has a double root at  $x = 3$ . **3**  
Find the values of  $a$  and  $b$ , where  $a$  and  $b$  are real numbers.

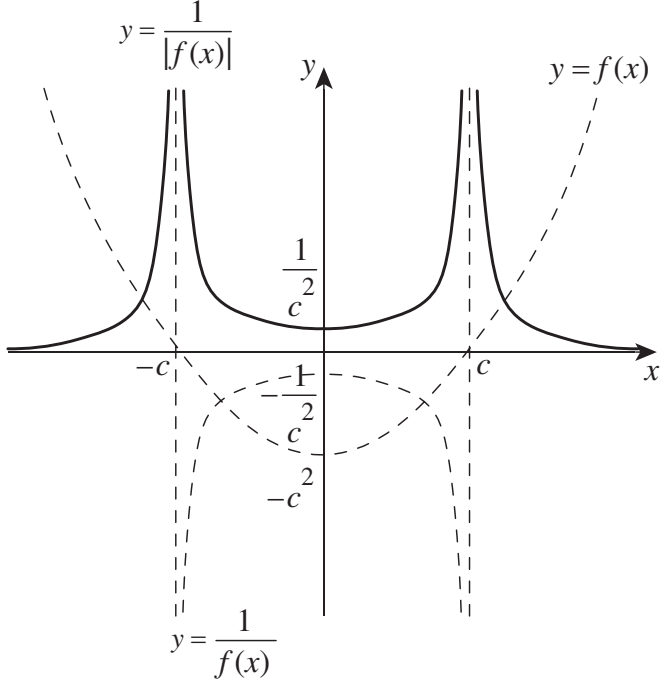
- (b) Find the values of  $n$  such that  $\binom{5n+3}{5n+1} \geq 528$ , where  $n$  is a positive integer. **3**

- (c) A large cylindrical water tank with a base radius of 0.4 m has a tap at the bottom. This tap allows water to flow out of the tank at a rate of  $\frac{dV}{dt} = k\sqrt{h}$ , where  $V \text{ m}^3$  is the volume of water,  $h$  is the depth of the water in metres,  $t$  is the time in minutes and  $k$  is a constant.

Initially, the water in the tank is 1 m deep. Twenty minutes after the tap has been turned on, the water in the tank is 0.36 m deep.

- (i) Show that  $k = -\frac{4}{625}\pi$ . **3**
- (ii) How long will it take for the tank to empty, correct to the nearest minute? **1**
- (d) Consider the expansion of  $(2x - p)^9$ . The coefficient of the fourth term is  $-672\,000$ . **2**  
Find the value of  $p$ .

- (e) Consider the function  $f(x) = x^2 - c^2$ , where  $c$  is a positive real number. **3**  
Sketch the graph of  $y = \frac{1}{|f(x)|}$ , showing all important features including the turning point(s), intercept(s) and asymptote(s).

| Sample answer  | Syllabus content, outcomes, targeted performance bands and marking guide   |
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| <p>(ii) <math>0.32\pi\sqrt{h} = -\frac{4}{625}\pi t + 0.32\pi</math></p> <p>When <math>h = 0</math>:</p> $0.32\pi\sqrt{0} = -\frac{4}{625}\pi t + 0.32\pi$ $-\frac{4}{625}\pi t + 0.32\pi = 0$ $\frac{4}{625}\pi t = 0.32\pi$ $t = 50 \text{ minutes}$ <p><i>Note: Consequential on answer to Question 11(c)(i).</i></p>   | <p>ME–C1 Rates of Change<br/>ME11–4 Bands E2–E3</p> <ul style="list-style-type: none"> <li>Provides the correct solution . . . . . 1</li> </ul>  |
| <p>(d) <math>T_4 = \binom{9}{3}(2x)^6(-p)^3</math></p> $= \binom{9}{3} \times 64x^6 \times -p^3$ $= -5376p^3x^6$ $-5376p^3 = -672\,000$ $p^3 = 125$ $p = 5$  | <p>ME–A1 Working with Combinatorics<br/>ME11–5 Bands E2–E3</p> <ul style="list-style-type: none"> <li>Provides the correct solution . . . . . 2</li> </ul> <hr/> <ul style="list-style-type: none"> <li>Provides the correct solution of <math>T_4</math> . . . . . 1</li> </ul>   |
| <p>(e)</p>  <p>The graph shows two functions on a Cartesian coordinate system. The x-axis has vertical asymptotes at <math>x = -c</math> and <math>x = c</math>, indicated by dashed lines. The function <math>y = \frac{1}{ f(x) }</math> is a solid curve that is U-shaped, with a minimum at <math>y = \frac{1}{c^2}</math> on the y-axis. The function <math>y = f(x)</math> is a dashed curve that is inverted U-shaped, with a maximum at <math>y = -\frac{1}{c^2}</math> on the y-axis. The x-axis also has a mark at <math>-c^2</math>.</p> | <p>ME–F1 Further Work with Functions<br/>ME11–1, 11–7 Bands E2–E3</p> <ul style="list-style-type: none"> <li>Provides the correct solution . . . . . 3</li> </ul> <hr/> <ul style="list-style-type: none"> <li>Draws the graph of <math>\frac{1}{ f(x) }</math> without turning points<br/>OR without asymptotes.</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>Draws the graph of <math>\frac{1}{f(x)}</math> with turning points<br/>AND asymptotes . . . . . 2</li> </ul> <hr/> <ul style="list-style-type: none"> <li>Draws the graph of <math>\frac{1}{f(x)}</math> without turning points<br/>OR without asymptotes . . . . . 1</li> </ul> |